Abstract—In this paper, we consider a source, which communicates to destination, with cooperation of a relay node. An eavesdropper is tapping the second hop, when all the links undergo Rayleigh fading. Relay nodes are the market players, who compete to trade their power to source in order to maximize their incentives. We evaluate the secrecy outage probability of a dual-hop decode-and-forward (DF) threshold-based cooperative relay network. Without assuming that all the relays can always perfectly decode, here we consider that only those relays who satisfy predetermined threshold, can correctly decode the message. We evaluate the utility of relays for both linear/non-linear and static/dynamic Cournot duopoly asymmetric model. We have shown that increase in channel gain from relay to eavesdropper, decreases the profit of relays and can have a dramatic effect on the Nash outcome.

Keywords—cournot game; decode-forward relay; outage probability; Nash equilibrium; threshold-based

I. INTRODUCTION

The broadcast nature of the wireless medium has given rise to many security issues [1], [2]. In information-theoretic security, to provide an additional protection to the traditional encryption techniques, there has been an upsurge of research interests, following the pioneering work of [3]. In systems where direct communication from source to destination is not possible due to power limitations, cooperating nodes receive the message from source and forward it to the destination [4]. These cooperative networks are susceptible to eavesdropping and to ensure a reliable communication, the application of security issues in the cooperative networks has received critical attention in the literature [4], [5]. Here, we consider such a cooperative and secure communication network as a means to trade power in mobile and wireless communications for local power efficiency and reliability.

In the communication network, cooperative relays play a significant role to save the uplink transmit power of mobile terminals and improve involved network performances, especially in the areas where signal strength is poor due to geographic referenced constraints [4], [5]. Physical layer security (PLS), or, information-theoretic security has become a fundamental approach for achieving security in wireless communication with confidential message delivery between source and destination [6]. Wyners pioneering work [3], have proved that a positive secrecy capacity can be achieved, when the main (or legitimate) channel is better than the wiretap (or eavesdropper) channel . Recently, to improve security in network game theoretic approaches have been proposed in [7]. Game theory is the mathematical tool for the study of optimization in situations of strategic interaction between two or more players, who chooses a strategy to play from their set of possible strategies and their payoff is decided by the combination of strategies played by all players involved. The game reaches equilibrium, when none of the player can profit by unilaterally deviating from their optimal strategy, while the other players keep their unchanged [8], [9]. Oligopoly market strategic theory deals with the competition between two or more firms as investigated in [8]–[10]. In [11], authors propose an auction based relay power allocation scheme over multi-user relay networks from the energy-efficient perspective. The source node which is in need of assistance of a relay node to communicate with its destination due to limited power, broadcasts a cooperation request to its neighbors [11]. For dual-hop transmission, the fixed relay network is modeled as a virtual oligopoly market in [12], and the the resource allocation problem is exploited using Cournot game model. The authors in [13] have presented a business model using sealed bid procurement auction based game theory for power-trading in cooperative wireless communication. The quality of service (QoS) constraints are taken in terms of the maximum bit error probability and the maximum acceptable delay in relaying the data. However, this work does not consider security aspect of communication, nor does it study the market scenario with respect to duopoly form of games.

In the dynamic Cournot scenario, the output is produced in the first period by one firm, while the other firm produces output in the second period. Stackelberg identified the sub-game perfect Nash equilibrium (SPNE) for this model and examined how dynamic considerations can give first mover a strategic advantage over the other [10]. A Stackelberg game model for cooperative cognitive radio network with active SUs is considered in [14]. However, [14] does not takes into account the secrecy constrains of the network. In contrast to the above study, our work is motivated to exploit static/dynamic pricing models for power trading in a threshold-based cooperative
relay network, in the presence of an eavesdropper. In the existing literature, typically it is assumed that due to high SNR scenario [15], a relay can correctly decode the message. However, this is not always a practical assumption as fading might degrade the signal strength, such that the relay is not able to correctly decode the message [16]. Correct decoding over a particular threshold SNR is thus a better assumption and is considered in this paper.

The remainder of this study is organized as follows. The system model is described in Section II. Outage probability expression are evaluated for threshold-based single cooperative relay system in Section III. In Section IV, using the expression we examine the market strategic game framework, where relay nodes use duopoly models under static/dynamic scenario, to trade their power to source. We also investigate how gains of the eavesdropper’s link, affect the utility of relays in this market game, which in turn motivates relay to keep low outage probability to earn high incentives. In Section V, simulation and numerical results of different duopoly models are discussed and finally, Section VI gives the concluding remarks.

II. SYSTEM MODEL

We consider the system model, consisting of a source $S$, a destination $D$, an eavesdropper $E$ and two DF relays $R_i$, $i \in [1, 2]$ which work in a dual-hop mode as depicted in the Fig.1. We have derived the expression for secrecy outage probability of this dual-hop DF threshold-based cooperative relay network. Threshold-based relaying is taken into account, where without assuming that all the relays can always perfectly decode, we consider that only those relays who meets predetermined threshold [16], illustrated as $\gamma_{th}$ for $S-R_i$ link can correctly decode the message [15], [17], [18]. Relays can perfectly decode the message from source only if the $S-R_i$ link SNR, $\Gamma_{sr_i}$, is greater than $\gamma_{th}$.

The links between various nodes works in half-duplex mode and are modeled as flat Rayleigh flat fading channels, which are mutually independent but not identical. The ICSI of the main channel, as well as, of the eavesdropper channel is are mutually independent but not identical. The ICSI of the eavesdropper’s link, affect the utility of relays in this market game, which in turn motivates relay to keep low outage probability to earn high incentives. In Section V, simulation and numerical results of different duopoly models are discussed and finally, Section VI gives the concluding remarks.

and corresponding cumulative distribution function (CDF) is given by

$$F_X(z) = 1 - e^{-z\beta_{xy}}.$$  \hspace{1cm} (3)

The $S-R_i$ channels $h_{sr_i}$, $R_i - D$ channels $h_{r_id}$ and $R_i - E$ channels $h_{r_id}$, $\forall i \in [1, 2]$ , are slowly varying Rayleigh flat fading channels [22]. Let $P_s$ and $P_r$, denote the average powers used at source and relay $R_i$ respectively. Also, let $N_{sr_i}, N_{r_id}$ and $N_{r_id}$ denote the variances of additive white Gaussian noise of $S - R_i$, $R_i - D$ and $R_i - E$ links respectively. The SNRs $\Gamma_{sr_i}$, $\Gamma_{r_id}$ and $\Gamma_{r_id}$ are exponentially distributed given as

$$\Gamma_{sr_i} = \frac{P_s|h_{sr_i}|^2}{N_{sr_i}}, \quad \Gamma_{r_id} = \frac{P_r|h_{r_id}|^2}{N_{r_id}}$$

and

$$\Gamma_{r_id} = \frac{P_r|h_{r_id}|^2}{N_{r_id}}$$

with average values $1/\beta_{sr_i}$, $1/\beta_{r_id}$ and $1/\alpha_{r_id}$ respectively where $\beta_{sr_i}$, $\beta_{r_id}$ and $\alpha_{r_id}$ are the parameters of the exponential distribution. An outage event occurs when the instantaneous secrecy rate is lower than the required secrecy rate of the cooperative relay system, given as $R_s$ where, $R_s > 0$ and $\rho = 2^{2R_s}$. [17], [19]. We have used $\rho$ for direct mapping of required secrecy rate $R_s$, and the probability of successful occurrence of this outage event is called outage probability $P_o$, which is a key metric in evaluating the performance of physical-layer security [22]. Achievable secrecy rate is the difference of the main channel information rate and the eavesdropper channel information rate of the system given as [3], [16], [19]

$$C_s = \frac{1}{2} \left[ \log_2 \left( 1 + \frac{\Gamma_M}{1 + \Gamma_E} \right) + \log_2 \left( 1 + \frac{\Gamma_{r_id}}{1 + \Gamma_{r_id}} \right) \right]$$

where $\Gamma_M = \Gamma_{r_id}$ is the SNR of the main link at $D$ and $\Gamma_E = \Gamma_{r_id}$ is the SNR of the eavesdropper link at $E$. The term $1/2$ here denotes that to complete this dual-hop transmission process, two time phase are required. The message transmitted by the source is decoded at the relay, whose threshold is satisfied in the first phase. In the second phase, one of the relay is selected to re-encode and forward the message to the destination. From (6), when the relay node does not meet the predetermined threshold due to shadow fading [23], no relay is selected for communication.

III. OUTAGE PROBABILITY ANALYSIS OF SINGLE RELAY SYSTEM

This section deals with the evaluation of the expression for secrecy outage probability of Decode-and-forward (DF) threshold-based dual-hop cooperative relay network. Each scenario is divided into two cases where, in the first case case, we consider that the message is decoded successfully [15], [18], as the SNR at the relay node satisfies the predetermined threshold while, in the second case we consider that the SNR at the relay node does not meet the predetermined threshold. Using (1)-(4), we evaluate outage probability for single $i^{th}$ relay as

$$P_o(R_s) = \mathbb{P}[\Gamma_{sr_i} \geq \gamma_{th}] \mathbb{P}[C_s \leq R_s] + \mathbb{P}[\Gamma_{sr_i} < \gamma_{th}]$$

$$= \mathbb{P}[\Gamma_{sr_i} \geq \gamma_{th}] \mathbb{P} \left[ \frac{1}{2} \left[ \log_2 \left( 1 + \frac{\Gamma_{r_id}}{1 + \Gamma_{r_id}} \right) \right] \leq R_s \right]$$
game. For the asymptotic behavior analysis, when 1/\beta \to \infty in the balanced case, the outage probability of threshold-based single cooperative relay system in (5), is given as

\[ P_o(R_e) = \frac{\beta_{d} (\rho + \alpha_{r,e} (\rho - 1))}{\alpha_{r,e}} + \gamma_{th} \beta_{sr}. \]

We can interpret from (6) that secrecy outage probability is inversely proportional to 1/\beta and it tends to zero, when main channel SNR (1/\beta) tends to infinity. It is directly proportional to the required threshold \gamma_{th}, eavesdropper channel SNR (1/\alpha_{r,e}) and desired secrecy rate \gamma_e. Also, the power of 1/\beta in the denominator of (6), is equal to the diversity order, D [16], [19]. As no relay selection is considered, it is intuitive that \(D = 1\), for this single cooperative relay system.

IV. DUOPOLY GAME FORMULATION

This section deals with linear/non-linear and static/dynamic Cournot duopoly asymmetric model for the threshold-based cooperative relay network. The utility of the relay node will be affected by the outage probability under static or dynamic conditions. In all the schemes, these relay nodes are closely related in terms of their location and channel statistics. Therefore, as outage probability depends on the channel statistics of the relay nodes which are nearly same, \(P^1_o = P^2_o = P_o\) is considered.

A. Cournot Model With Linear Cost

In this Cournot linear asymmetric cost model, both the competitive relay nodes choose quantity as their strategic variable simultaneously, under the assumption of complete information, linear demand function and fixed cost per unit resource. The quantity \(Q_i\) produced by the two relay nodes, is taken as the quantity of their powers resources \(Q_1 = P_{w_1}\) and \(Q_2 = P_{w_2}\) respectively.

Let the price per unit power resource for the \(i^{th}\) relay be \(P_i\)

\[ P_1 = A(1 - P_o) - Q_1 - Q_2 \]
\[ P_2 = A(1 - P_o) - Q_2 - Q_1 \]

The generalized price per unit power resource is

\[ P_i = A(1 - P_o) - Q \]

where \(Q = \sum_{i=1}^{2} Q_i\), is the total quantity or the total market demand. This is an inverse demand function [9], where the price set by the relays will decrease with the increase in the demand of power resource by the source node. Also, it varies inversely with outage probability, which in turn motivates the relay nodes to maintain high secrecy capacity in order to maximize their incentives. \(A\) is the positive constant. Let the marginal cost be \(C_i\) for the relays, and \(C_i Q_i\), will be the total cost of power production for each relay. The utility of the \(i^{th}\) relay \(U_i\) is given as the difference between the total revenue and the total cost

\[ U_1 = (A(1 - P_o) - (Q_1 + Q_2))Q_1 - C_1Q_1 \]
\[ U_2 = (A(1 - P_o) - (Q_2 + Q_1))Q_2 - C_2Q_2 \]

The second-order condition of profit maximization holds, because the second derivative of the profit function for each relay is negative hence, best response \(Q^*_i\) of \(i^{th}\) relay using (10) and (11) is

\[ Q^*_1 = \frac{A(1 - P_o) - Q_1}{2} \]
\[ Q^*_2 = \frac{A(1 - P_o) - C_2 - Q_1}{2} \]

At Nash equilibrium both the best responses \(BR_i\) intersect

\[ Q^*_1 = BR_1(Q^*_2) \]
\[ Q^*_2 = BR_2(Q^*_1) \]

After substituting both equations, we get the Cournot Nash equilibrium solution as

\[ (Q^*_1, Q^*_2) = \left( \frac{A(1 - P_o) - 2C_1 + C_2}{3}, \frac{A(1 - P_o) - 2C_2 + C_1}{3} \right) \]

\[ (U^*_1, U^*_2) = \left( \frac{(A(1 - P_o) - 2C_1 + C_2)^2}{9}, \frac{(A(1 - P_o) - 2C_2 + C_1)^2}{9} \right) \]

B. Cournot Model With Non-Linear Cost

In this Cournot non-linear asymmetric cost model, let the marginal cost be \(C_i Q_i\) for the relays, and \(C_i^2 Q_i^2\), will be the total cost of power production for each relay. The utility function of the \(i^{th}\) relay \(U_i\) is given as the difference between
the total revenue and the total cost

\[ U_1 = (A(1 - P_o) - (Q_1 + Q_2))Q_1 - \frac{C_1 Q_1^2}{2} \]  
\[ U_2 = (A(1 - P_o) - (Q_2 + Q_1))Q_2 - \frac{C_2 Q_2^2}{2} \]  

The second-order condition of profit maximization holds, because the second derivative of the profit function for each relay is negative hence, best response is \( Q_i^* \) of \( i^{th} \) relay. At Nash equilibrium both the best responses \( BR_i \) intersect

\[ Q_1^* = BR_1(Q_2^*) \]  
\[ Q_2^* = BR_2(Q_1^*) \]  

After substituting both equations, we get the Cournot Nash equilibrium solution as

\[ (Q_1^*, Q_2^*) = \left( \frac{(1 + C_1)A(1 - P_o)}{(1 + C_1)(1 + C_2) + (2 + C_1 + C_2)}, \frac{(1 + C_2)A(1 - P_o)}{(1 + C_1)(1 + C_2) + (2 + C_1 + C_2)} \right) \]  

One modification is to introduce QoS performance parameter. The cost function is defined in such a manner that it increases with increase in degradation of QoS factor \( d_i \), which depend on the channel statistics. There will be degradation in QoS if the relay nodes do not satisfy the secrecy requirements of the source node such that secrecy outage occurs in the network. Let the marginal cost be \( C_i Q_i + d_i \) for the relays. The utility function of the \( i^{th} \) relay \( U_i \) is given as the difference between the total revenue and the total cost

\[ U_1 = (A(1 - P_o) - (Q_1 + Q_2))Q_1 - \frac{C_1 Q_1^2}{2} - d_1 Q_1 \]  
\[ U_2 = (A(1 - P_o) - (Q_2 + Q_1))Q_2 - \frac{C_2 Q_2^2}{2} - d_2 Q_2 \]

The second-order condition of profit maximization holds, because the second derivative of the profit function for each relay is negative hence, best response is \( Q_i^* \) of \( i^{th} \) relay. At Nash equilibrium both the best responses \( BR_i \) intersect

\[ Q_1^* = BR_1(Q_2^*) \]  
\[ Q_2^* = BR_2(Q_1^*) \]  

After substituting both equations, we get the Cournot Nash equilibrium solution as

\[ (Q_1^*, Q_2^*) = \left( \frac{(1 + C_1)A(1 - P_o) - (C_2 + 2)d_1 + d_2}{(1 + C_1)(1 + C_2) + (2 + C_1 + C_2)}, \frac{(1 + C_2)A(1 - P_o) - (C_1 + 2)d_2 + d_1}{(1 + C_1)(1 + C_2) + (2 + C_1 + C_2)} \right) \]  

From the above observation, we can examine that \( \frac{\partial Q_1^*}{\partial d_1} < 0 \) and \( \frac{\partial Q_1^*}{\partial d_2} > 0 \), concluding that our own degradation in QoS lowers the supply, but the opponent’s degradation in QoS increases the supply.

### C. Stakelberg Model

In this Stakelberg 2-period model, we investigate the dynamic Cournot leader-follower scenario, when the choice of strategic variable is quantity [10]. The relays move sequentially with rest all assumptions same as Cournot linear model. This decision is irreversible and cannot be changed in the second period. The utility function of the \( i^{th} \) relay is

\[ U_1 = (A(1 - P_o) - (Q_1 + Q_2))Q_1 - C_1 Q_1 \]  
\[ U_2 = (A(1 - P_o) - (Q_2 + Q_1))Q_2 - C_2 Q_2 \]

The best response \( Q_2^* \) of follower relay is given as

\[ Q_2^* = \frac{A(1 - P_o) - C_2 + C_1 - Q_1}{2} \]

The best response \( Q_1^* \) of leader relay, after knowing the best response of the follower relay \( Q_2^* \), is given as

\[ Q_1^* = \frac{A(1 - P_o) - 2C_1 + C_2}{2} \]

After substituting both equations, we get the dynamic Cournot Nash equilibrium solution as

\[ (Q_1^*, Q_2^*) = \left( \frac{A(1 - P_o) - 2C_1 + C_2}{2}, \frac{A(1 - P_o) - 2C_2 + C_1}{4} \right) \]

\[ (U_1^*, U_2^*) = \left( \frac{(A(1 - P_o) - 2C_1 + C_2)^2}{8}, \frac{(A(1 - P_o) - 2C_2 + C_1)^2}{16} \right) \]

Under this dynamic sequential asymmetric Cournot game, leader will always have a strategic advantage over others [10].

### D. Monopoly Model

In this Monopoly model, there is only single relay (monopolist), who decides on quantity \( Q \) and trades each unit of power at marginal cost \( C \). The market price, \( P \) is determined by inverse market demand \( Q \).

\[ P = A(1 - P_o) - Q \]

The relays profit function, if it produces \( Q \) units is

\[ U_1 = (A(1 - P_o) - Q)Q - C_1 Q \]

The relay seeks to maximize its profit by choosing \( Q \) such that

\[ Q_1^* = \frac{A(1 - P_o) - C_1}{2} \]

The equilibrium price \( P_1^* \) and utility \( U_1^* \) is given as

\[ P_1^* = \frac{A(1 - P_o) + C_1}{2} \]

\[ U_1^* = \frac{(A(1 - P_o) - C_1)^2}{4} \]
The relay earns high incentives when it has no other competition in the market.

V. NUMERICAL ANALYSIS

This section presents the analytical results of a threshold-based dual-hop DF cooperative relay network, that exactly matches with the simulation results. Noise power is assumed to be same at all the nodes. To cover feasible range of required secrecy rate, both low and high desired rate of $R_s = 0.1$ and $R_s = 2.0$ are considered.

Fig. 1 shows the outage probability $P_o(R_s)$ of single $i^{th}$ cooperative relay, as expressed in (5) with total SNR $1/\beta$. This figure has been plotted with different relay to eavesdropper average SNR $1/\alpha_{r,e} = 1/\alpha = 6, 9, 12$ dB, desired secrecy rate $R_s = 0.1, 1.0, 2.0$ and fixed $\gamma_{th} = 3$ dB. It is observed from the figure that increase in eavesdropper channel quality increases the outage probability of the system. Secrecy outage probability increases with increase in desired secrecy rate $R_s$. Also, the corresponding asymptotic analysis as given in (6) is depicted by straight solid lines crossing through the curves.

In Fig. 2, utility $U_i$ of single $i^{th}$ relay is plotted for the Cournot duopoly linear model as expressed in (17) with main link SNR $1/\beta$. The figure is plotted with different relay to eavesdropper average SNR $1/\alpha_{r,e} = 1/\alpha = 3, 6, 9, 12$ dB and $\gamma_{th} = 3, 6$ dB, while other constants are taken as $A = 19$, assuming marginal cost of production, $C_1 = 2$ and $C_2 = 1$. It is observed from the figure that improvement in eavesdropper channel quality degrades the utility of relay. It is interesting to observe that increase in $\gamma_{th}$ lowers the utility of relay, especially at low SNR values. This is because with increase in threshold $\gamma_{th}$, the probability of the relay to get selected for forwarding source data decreases, thus outage probability increases. It is also observed that the utility of relay node increases with increase in SNR of the main link, especially for lower values of SNR. This observation holds true for other duopoly economic models also.

In Fig. 3, comparison of total quantity of relays $Q$ for various duopoly economic models with $1/\alpha = 3, 6$ dB and $\gamma_{th} = 3$ dB.
linear scheme the marginal cost itself is a function of quantity, while it is constant for the linear scheme.

VI. CONCLUSION
In this paper, the secrecy outage probability of DF cooperative threshold-based relay system is evaluated, without assuming that all the relays can always perfectly decode. We observe that desired secrecy rate, eavesdropper channel quality and predetermined threshold has a significant impact on outage performance and utility of the system. We also observe that under comparative analysis of various economic duopoly power trading models, the relays will be better off if the strategy of its opponent is known.

VII. FUTURE WORK
This work can be extended for multi-eavesdropper scenario. The secrecy performance of the system can be then analyzed with various economic duopoly power trading models.

REFERENCES